The OPAL Equation of State and Low Metallicity Isochrones

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ABSTRACT

The Yale stellar evolution code has been modified to use the OPAL equation of state tables (Rogers 1994). Stellar models and isochrones were constructed for low metallicity systems ($-2.8 \leq$ [Fe/H] ≤ -0.6). Above $M \sim 0.7\,M_\odot$, the isochrones are very similar to those which are constructed using an equation of state which includes the analytical Debye-Hückel correction at high temperatures. The absolute magnitude of the main sequence turn-off (M_v(TO)) with the OPAL or Debye-Hückel isochrones is about 0.06 magnitudes fainter, at a given age, than M_v(TO) derived from isochrones which do not include the Debye-Hückel correction. As a consequence, globular clusters ages derived using M_v(TO) are reduced by 6 – 7% as compared to the ages determined from the standard isochrones. Below $M \sim 0.7\,M_\odot$, the OPAL isochrones are systematically hotter (by approximately 0.04 in B–V) at a given magnitude as compared to the standard, or Debye-Hückel isochrones. However, the lower mass models fall out of the OPAL table range, and this could be the cause of the differences in the location of the lower main-sequences.

Subject headings: globular clusters: general – stars: interiors – stars: evolution

1. Introduction

Low heavy element abundances and a spherical distribution about the galactic centre are clear indicators that globular clusters were among the first objects formed in our galaxy. Consequently it is important to determine the ages of globular clusters to gain an understanding of how our galaxy formed and to provide an estimate for the age of the universe. The ages of globular clusters are derived by comparing stellar evolution models to observations of globular cluster colour magnitude diagrams. Stellar models are constructed by solving the basic stellar structure equations. The solution of the stellar structure equations require that the opacity, nuclear reaction rates and equation of state (hereafter EOS) be specified. Stellar evolution models are a good representation of the true evolution of a star only if the input physics used in the solution of the stellar structure equations are accurate. As new calculations for the opacity, reaction rates or EOS become available, it is important to assess the impact of these improvements on stellar models. Iglesias & Rogers (1991) calculated a new set of opacities which were considerably different (in certain temperature-density regimes) from previous calculations. These differences helped to solve many long standing problems in stellar astrophysics. The new opacities did not appreciably alter the ages of the globular clusters, which are best derived using the absolute magnitude of the main sequence turn-off (M_v(TO), Chaboyer 1995). Recently Rogers (1994) has made available EOS tables appropriate for stellar conditions. This is the same EOS (hereafter the OPAL EOS) which was used in the construction of the Iglesias & Rogers (1991) opacities. In order to investigate the effects on the OPAL EOS on age estimates for globular clusters, the Yale stellar evolution code has been modified to incorporate the OPAL EOS, and low metallicity stellar models and isochrones have been constructed.

The importance of the EOS for stellar evolution calculations has long been recognized. Eggelton, Faulkner & Flannery (1973) developed an EOS that has been widely used in stellar evolution calculations, though not in the Yale code. At high temperatures (above 10^6 K) the standard EOS in the Yale stellar evolution code is the ideal gas law, with radiation pressure and degeneracy effects added in the standard manner (eg. Schwarzschild 1958). At low temperatures a Saha equation (Saha 1920) which includes the single ionization of hydrogen, the first ionization of the metals and both ionizations of helium is used as the EOS. This EOS results in typical globular clusters ages of 15 Gyr. A modification of this standard EOS includes the effect of Coulomb forces by using the Debye-Hückel correction. Guenther *et al.* (1992) describe the implementation of the Debye-Hückel correction into the Yale stellar evolution code. The analytical Debye-Hückel correction implemented in the Yale code is only valid for a fully ionized plasma, so the Saha equation is still used below 10^6 K. Isochrones constructed using the Debye-Hückel correction yield globular cluster ages which are approximately 1 Gyr younger than the standard isochrones. However, the Debye-Hückel correction is only valid when

$$\Lambda \equiv \frac{\sqrt{4\pi} Z^3 e^3 \rho^{1/2}}{(kT)^{3/2}} < 0.2 \qquad (1)$$

(see Rogers 1994 for a full discussion). This criterion is often violated within a stellar plasma. Thus, it is not clear if including the Debye-Hückel correction actually results in an improved EOS.

The OPAL EOS includes corrections for Coulomb forces which are valid over the entire range of temperatures and densities encountered in standard stellar models, and so represents a considerable improvement over the analytical Debye-Hückel correction. In §2 the construction and evolution of the stellar models is described, while §3 discusses the resulting isochrones. The implications that these isochrones have for globular cluster ages is described in §4. A brief summary is presented in §5.

2. Stellar Models

The OPAL EOS tables were integrated into the Yale stellar evolution code using the interpolation routines supplied by Rogers. Stellar models (and isochrones) were then constructed using 3 different EOS: (1) the standard EOS described in the introduction; (2) the standard EOS with the analytical Debye-Hückel correction at high temperatures, and (3) the OPAL EOS. For each EOS, a series of stellar models with masses ranging from $M = 0.5\ M_\odot$ to $\sim 1.0\ M_\odot$ (in $0.05\,M_\odot$ increments) were evolved from the zero-age main sequence to the sub-giant branch. Numerical difficulties related to the OPAL EOS prevented us from evolving models on the giant branch. The numerical tolerances, and input physics (aside from the EOS) were identical for all evolutionary runs. In order to span the range of metallicities encountered in the globular cluster system, models were evolved with $Z = 6\times10^{-5}$, 2×10^{-4}, 6×10^{-4}, 2×10^{-3}, 4×10^{-3}, and 7×10^{-3}, corresponding to [Fe/H] $= -2.8;\ -2.3;\ -1.8;\ -1.3$ all with $[\alpha/\mathrm{Fe}] = +0.40$; [Fe/H] $= -0.9$, $[\alpha/\mathrm{Fe}] = +0.30$;

and [Fe/H] = −0.6, [α/Fe] = +0.20. The models use a scaled solar composition, and the effect of α-element enhancement has been taken into account by modifying the relationship between Z and [Fe/H], as prescribed by Salaris, Chieffi & Straniero (1993).

All models included the effects of ^{4}He diffusion, using the diffusion coefficients of Michaud & Proffitt (1993). A mixing length of $\alpha = 1.7$ was used in all of the models. The various EOS require mixing lengths which differ by less than 2% from each other for a solar calibrated model, so it was not deemed necessary to use different mixing lengths in the low metallicity models. The other input physics used in the models are the standard ones for the Yale code, as described by Chaboyer (1995).

The effect of the OPAL EOS on the structure of the stellar models is illustrated in Figure 1a, which compares the run of temperature and density for $M = 0.5$ and $0.8\,M_\odot$ stellar models using the OPAL and Debye-Hückel EOS. The differences between the two EOS are small. Unfortunately, the $M = 0.5\,M_\odot$ model falls outside the OPAL EOS around $\log T = 5$. When the models fall outside the OPAL tables, the standard EOS has been substituted. Figure 1b plots the difference in density ($\Delta\rho/\rho$) as a function of temperature, where the pressure and temperature have been taken from the Debye-Hückel models. We see that for a given temperature and pressure the OPAL EOS gives densities which typically differ by $2-4\%$ from the Debye-Hückel case. This plot also makes it clear where the $M = 0.5\,M_\odot$ model falls outside the table, for these regions are identified by the solid line at $\Delta\rho/\rho = 0$ in Figure 1b. We have made similar plots for other masses, and stages of evolution, and have determined that all models with $M \gtrsim 0.75\,M_\odot$ stay within the OPAL EOS tables throughout their evolution. Lower mass models fall outside the tables near the zero-age main sequence (ZAMS), but are usually entirely within the OPAL tables as they evolve off of the ZAMS. Unfortunately, for low mass models ($M \lesssim 0.7\,M_\odot$), it is the ZAMS models which are most important, as such low mass stars are still essentially unevolved, even in globular clusters. For this reason, we must view with caution properties of the OPAL EOS models and isochrones below $M \simeq 0.7\,M_\odot$, for they may be affected by the fact that the EOS abruptly changes to the standard one in part of the model.

The effects of the Debye-Hückel and OPAL EOS on stellar evolutionary tracks is shown in Figure 2 ($M = 0.6\,M_\odot$) and Figure 3 ($M = 0.8\,M_\odot$). Figure 2 demonstrates that the low mass OPAL models are shifted to hotter effective temperatures near the ZAMS as compared to the Debye-Hückel models. By the time the models reach the turn-off (and the model is entirely within the OPAL table), the OPAL and Debye-Hückel evolutionary tracks are nearly identical. This suggests that the shift near the ZAMS may be due to the fact that the model is falling outside the OPAL table around $\log T = 5$. Figure 3 shows that for higher mass models, the evolution is nearly identical for the OPAL and Debye-Hückel cases. Thus, the turn-off properties of isochrones constructed from the OPAL or Debye-Hückel models should be similar. In order to demonstrate the possible effect that falling out of the OPAL tables has on the low mass models, Figure 3 plots a model in which the OPAL EOS has been used except for $\log T = 4.4 - 5.2$, in which case the standard EOS was substituted. Note that this model is shifted in cooler effective temperature as compared to the pure OPAL, or Debye-Hückel cases. This shift is in the opposite direction than that which occurs in the low mass models. However, it does demonstrate that evolutionary tracks can be substantially modified if the OPAL EOS is not used in the entire model. Thus, the shift in effective temperature found in the low mass OPAL models may be due to the fact that these models fall outside the OPAL EOS tables around $\log T = 5$.

3. Isochrones

Isochrones were constructed by interpolating the evolutionary tracks using the method of equal evolutionary points (Prather 1976) for the different Z values and EOS discussed above. The isochrones spanned the age range 8 − 22 Gyr, in 1 Gyr increments. The colour transformation of Green, Demarque & King (1987) was used to transform from the theoretical luminosities and temperatures to the UBVRI system. Figure 4 plots the 14 Gyr, [Fe/H] = −1.3 isochrones for the different EOS. Brighter than $M_V \simeq 5.2$, the Debye-Hückel and OPAL isochrones are very similar. They are somewhat redder, and have a fainter main sequence turn-off magnitude as compared to the standard isochrones. Fainter than $M_V \simeq 5.2$, the standard and Debye-Hückel isochrones are very similar. The OPAL isochrones are somewhat hotter (at a given magnitude) as compared to the other isochrones along the lower main sequence. The above statements are true for all of the isochrones we have constructed. The isochrones are always quite similar around $M_V = 5.2$. This corresponds to a mass of roughly $0.7M_\odot$. This critical mass is a function of the metallicity and age of the isochrone, and the varies between 0.67 and $0.74\,M_\odot$. This is a similar mass range to where the stellar models fall out of the

OPAL EOS tables (around $\log T = 5$). As discussed in the previous section, the change in location of the lower main sequence in the OPAL isochrones (as compared to the standard or Debye-Hückel isochrones) is likely due to the fact that we have been forced to use the standard EOS around $\log T = 5$ for the lower mass models.

In order to determine which set of isochrones most closely match the observations of globular clusters, we have fit the isochrones to the fiducial sequences of M92 (Stetson & Harris 1988), and NGC 288 (Bolte 1992). The fit to M92 (with [Fe/H] $= -2.3$) is shown in Figure 5. The best fit to the data was found using the objective near point estimator technique of Flannery & Johnson (1982; see also Durrell & Harris 1993). In performing this fit, the distance modulus was allowed to vary by ± 0.1 mag from its nominal value of 14.52, and the reddening to vary between 0.0 and 0.05 (its nominal value is 0.02). For all three sets of isochrones, the best fit was found with an age of 17 Gyr. The standard isochrones are best able to reproduce the shape of the observed CMD. The Debye-Hückel isochrones (not shown) produced a fit which was nearly as good. The OPAL isochrones do not fit the data as well; this is due to the fact that the slope of the lower main sequence ($\Delta V/\Delta(B-V)$) is too steep in the OPAL isochrones. Although not shown here, very similar results are found for the fit to NGC 288.

The fact that the OPAL isochrones do not fit the observations as well as the standard, or Debye-Hückel isochrones does not imply that the OPAL EOS is inferior to the standard or Debye-Hückel EOS. The shape of the isochrones is influenced by our colour calibration, choice of mixing length, surface boundary conditions, as well as the fact that the lower mass models fall out of the OPAL EOS tables. All that may be concluded from Figure 5 is that for our choice of input physics, the standard isochrones are best able to reproduce the morphology of observed colour-magnitude diagrams.

4. Globular Cluster Ages

Fitting of isochrones to observed colour magnitude diagrams is subject to numerous theoretical uncertainties (colour calibration, mixing length theory, model atmospheres, etc). For this reason, it is best to determine the ages of globular clusters using the absolute magnitude of the main sequence turn-off ($\mathrm{M_v(TO)}$). This luminosity based age indicator is subject to far fewer theoretical uncertainties than age indicators which use colour information. For each set of isochrones, $\mathrm{M_v(TO)}$ was determined by taking the average magnitude of the bluest point on the isochrones. For each metallicity, the age in Gyr (t_9) was fit as a function of $\mathrm{M_v(TO)}$ using an equation of the form:

$$t_9 = \beta_0 + \beta_1 \mathrm{M_v(TO)} + \beta_2 \mathrm{M_v(TO)}^2. \qquad (2)$$

This simple quadratic fit did an excellent job of reproducing the mean trend in the calculated $\mathrm{M_v(TO)}$, while removing some of the 'wiggles' introduced by the isochrone construction program. Table 1 give the coefficients of the fit for the standard and OPAL isochrones. As Figure 4 demonstrates, $\mathrm{M_v(TO)}$ for the Debye-Hückel and OPAL isochrones are very similar, so the fit coefficients for the Debye-Hückel isochrones have not been included in Table 1.

Figure 6 plots the fit of age as a function of $\mathrm{M_v(TO)}$ for the [Fe/H] $= -1.3$ isochrones. This plot demonstrates the similarity in $\mathrm{M_v(TO)}$ for the Debye-Hückel and OPAL isochrones. For a given value of $\mathrm{M_v(TO)}$ the standard isochrones imply ages that are approximately $0.5 - 1.5$ Gyr older than the OPAL or Debye-Hückel isochrones. It is worthwhile noting that the greatest age reduction occurs for the oldest isochrones. The trends seen here are true for all metallicities that have been calculated. Thus, globular cluster ages determined using $\mathrm{M_v(TO)}$ will be approximately 1 Gyr younger with the Debye-Hückel or OPAL isochrones, as compared to the standard isochrones.

In order to apply our isochrones to actual data, it is necessary to convert observed magnitudes into absolute magnitudes. This may be accomplished in an reddening independent way by comparing the difference between $\mathrm{M_v(TO)}$ and the magnitude of the horizontal branch (in the RR Lyr instability strip). This age determination technique is referred to as $\Delta\mathrm{V(TO-HB)}$. In order to use $\Delta\mathrm{V(TO-HB)}$ to determine the ages, we must specify the absolute magnitude of the RR Lyr stars ($M_V(\mathrm{RR})$). There are a number of different ways to determine $M_V(\mathrm{RR})$, all of which find that $M_V(\mathrm{RR}) = \mu\,[\mathrm{Fe/H}] + \gamma$ where μ is the slope with metallicity, and γ is the zero-point. Estimates for the slope vary from 0.15 to 0.30. For this study, a value of $\mu = 0.20$ has been chosen, which is in reasonable agreement with Baade-Wesslink and infrared flux methods for field RR Lyr stars (Carney, Storm & Jones 1992; Skillen *et al.* 1993), and theoretical calculations (Lee 1990). For the zero-point, a value of $\gamma = 0.94$ has been chosen. This value is simply the average of the values determined by Walker (1992) who observed LMC RR Lyrs (using the SN1987A distance to the LMC) and Layden *et al.* (1995) who determined $M_V(\mathrm{RR})$ using statistical parallax obser-

TABLE 1
MODELING COEFFICIENTS

[Fe/H]	Standard β_1	Standard β_2	Standard β_3	OPAL β_1	OPAL β_2	OPAL β_3
−2.82	60.595	−38.353	6.945	61.079	−37.933	6.730
−2.29	72.007	−43.525	7.319	85.839	−50.317	8.088
−1.82	73.431	−43.722	7.160	79.331	−45.804	7.252
−1.29	101.546	−56.583	8.469	84.699	−47.586	7.240
−0.91	96.273	−53.127	7.873	94.421	−51.422	7.532
−0.59	93.204	−51.140	7.531	89.324	−48.428	7.071

vations. Thus, the following formulae was used to determine the absolute magnitude of the RR Lyr stars:

$$M_V(\mathrm{RR}) = 0.20\,[\mathrm{Fe/H}] + 0.94. \qquad (3)$$

The $\mathrm{M_v}$(TO) values determined from the isochrones are combined with equation (3) to determine ΔV(TO−HB) as a function of metallicity and age (in Gyr). The resulting grid was modeled using an equation of the form

$$\begin{aligned} t_9 &= a_o + a_1\Delta\mathrm{V} + a_2\Delta\mathrm{V}^2 + a_3[\mathrm{Fe/H}] \\ &+ a_4[\mathrm{Fe/H}]^2 + a_5\Delta\mathrm{V}[\mathrm{Fe/H}] \qquad (4) \end{aligned}$$

and globular cluster ages were determined using the above equation for each EOS. Table 2 lists the ages for 40 globular clusters with observed ΔV(TO − HB) values which are within the metallicity range of our set of isochrones. The observed ΔV(TO − HB) are taken primarily from the compilation of Chaboyer, Sarajedini & Demarque (1992). A few measurements are from the compilation of Walker (1992). The observed ΔV(TO−HB) for NGC 6535 is from Sarajedini (1994), and that of NGC 6652 is from Ortolani, Bica & Barbuy (1994). The cluster metallicities are taken primarily from Zinn & West (1984), with a few exceptions as noted by Chaboyer *et al.* (1992). The error in the derived age was determined by propagating through the errors in ΔV(TO−HB) and [Fe/H] as determined by the observers. The average age of the globular clusters is 13.90 Gyr for the standard isochrones, 13.03 Gyr for the Debye-Hückel isochrones and 12.97 Gyr for the OPAL isochrones. Thus, as expected, the OPAL EOS results in ages similar to the Debye-Hückel EOS, both of which about 0.9 Gyr younger than the standard isochrones. Note that there is evidence for an age range within the globular clusters, so the average ages quoted above should not be taken as 'the' age of the globular clusters. The difference in age as determined by the OPAL and standard isochrones is plotted as a function of metallicity and age in Figure 7. It is clear that the largest age reductions occur for the oldest clusters. The age reduction is not as well correlated with metallicity, although the largest age reductions tend to occur for the most metal-poor clusters; this is likely due to the fact that the metal-poor clusters also tend to be the oldest clusters. As the zero-point of our ages is set by the zero-point in the M_V(RR) relation (eq. 3), larger age reductions will occur if a brighter zero-point for M_V(RR) is adopted.

5. Summary

The OPAL EOS tables from Rogers (1994) have been integrated into the Yale stellar evolution code. Stellar models and isochrones were constructed with the standard EOS, an equation of state which includes the Debye-Hückel correction at high temperatures, and the OPAL EOS. These calculations covered a range in metallicity ($-2.8 \leq$ [Fe/H] $- 0.6$) and age (8 − 22 Gyr) appropriate for the study of globular clusters. Lower mass models (below $M \lesssim 0.7\,M_\odot$) with the OPAL EOS are shifted to hotter effective temperatures as compared to the standard or Debye-Hückel models. However, the lower mass models fall out of the OPAL EOS tables around $\log T = 5$. A test was performed whereby a $0.8\,M_\odot$ stellar model (which is normally entirely within the OPAL EOS tables) was evolved with the OPAL EOS except near $\log T = 5$, where the standard EOS was substituted. This evolutionary track was shifted to cooler effective temperature. Although the shift was in the opposite direction from that found in the lower mass models, it nevertheless points out that the cause of the shift in the H-R diagram for lower mass stars could be due to

TABLE 2
GLOBULAR CLUSTER AGES

NGC	Name	[Fe/H]	ΔV	Standard Age (Gyr)	Debye-Hückel Age (Gyr)	OPAL Age (Gyr)
104	47 Tuc	-0.71 ± 0.08	3.61 ± 0.10	14.29 ± 1.9	13.20 ± 1.7	13.23 ± 1.7
288		-1.40 ± 0.12	3.60 ± 0.12	14.82 ± 1.9	13.91 ± 1.7	13.83 ± 1.7
362		-1.27 ± 0.07	3.42 ± 0.14	12.41 ± 2.4	11.59 ± 2.2	11.55 ± 2.2
1261		-1.31 ± 0.09	3.44 ± 0.12	12.40 ± 1.6	11.68 ± 1.5	11.62 ± 1.5
1851		-1.36 ± 0.09	3.45 ± 0.10	12.58 ± 1.4	11.85 ± 1.2	11.79 ± 1.2
1904	M79	-1.69 ± 0.09	3.45 ± 0.12	13.01 ± 1.7	12.23 ± 1.5	12.16 ± 1.5
2298		-1.85 ± 0.11	3.49 ± 0.21	13.84 ± 3.0	12.97 ± 2.8	12.90 ± 2.8
2808		-1.37 ± 0.09	3.50 ± 0.14	13.29 ± 2.0	12.50 ± 1.8	12.43 ± 1.8
3201		-1.61 ± 0.12	3.39 ± 0.17	12.10 ± 2.2	11.40 ± 2.0	11.34 ± 2.0
4147		-1.80 ± 0.26	3.60 ± 0.12	15.43 ± 2.0	14.44 ± 1.8	14.36 ± 1.8
4590	M68	-2.09 ± 0.11	3.42 ± 0.10	13.30 ± 1.4	12.47 ± 1.3	12.41 ± 1.3
5024	M53	-2.04 ± 0.08	3.56 ± 0.14	15.24 ± 2.2	14.26 ± 2.0	14.19 ± 2.0
5053		-2.41 ± 0.06	3.48 ± 0.12	14.84 ± 1.8	13.88 ± 1.6	13.83 ± 1.6
5272	M3	-1.66 ± 0.06	3.54 ± 0.09	14.26 ± 1.4	13.38 ± 1.2	13.30 ± 1.2
5466		-2.22 ± 0.36	3.58 ± 0.12	15.94 ± 2.1	14.89 ± 1.9	14.83 ± 1.9
5897		-1.68 ± 0.11	3.60 ± 0.18	15.23 ± 2.9	14.26 ± 2.6	14.18 ± 2.6
5904	M5	-1.40 ± 0.06	3.49 ± 0.11	13.18 ± 1.6	12.40 ± 1.4	12.33 ± 1.4
6101		-1.81 ± 0.15	3.40 ± 0.12	12.53 ± 1.6	11.78 ± 1.5	11.72 ± 1.4
6121	M4	-1.33 ± 0.10	3.55 ± 0.16	13.97 ± 2.4	13.13 ± 2.2	13.06 ± 2.2
6171	M107	-0.99 ± 0.06	3.60 ± 0.18	14.40 ± 2.8	13.55 ± 2.6	13.50 ± 2.5
6205	M13	-1.65 ± 0.06	3.55 ± 0.21	14.40 ± 3.2	13.50 ± 2.9	13.43 ± 2.9
6218	M12	-1.34 ± 0.09	3.45 ± 0.14	12.56 ± 1.9	11.83 ± 1.7	11.77 ± 1.7
6254	M10	-1.75 ± 0.08	3.75 ± 0.15	17.92 ± 2.7	16.73 ± 2.5	16.65 ± 2.5
6341	M92	-2.24 ± 0.08	3.65 ± 0.12	17.15 ± 2.1	16.00 ± 1.9	15.94 ± 1.9
6397		-1.91 ± 0.14	3.64 ± 0.14	16.29 ± 2.4	15.22 ± 2.2	15.15 ± 2.2
6584		-1.54 ± 0.15	3.47 ± 0.12	13.08 ± 1.7	12.29 ± 1.5	12.23 ± 1.5
6535		-1.75 ± 0.15	3.66 ± 0.19	16.33 ± 3.2	15.27 ± 3.0	15.19 ± 3.0
6652		-0.89 ± 0.15	3.35 ± 0.16	10.28 ± 2.3	9.52 ± 2.1	9.53 ± 2.1
6752		-1.54 ± 0.09	3.65 ± 0.16	15.82 ± 2.7	14.82 ± 2.4	14.74 ± 2.4
6809	M55	-1.82 ± 0.15	3.55 ± 0.10	14.68 ± 1.6	13.75 ± 1.4	13.67 ± 1.4
7006		-1.59 ± 0.07	3.55 ± 0.12	14.31 ± 1.8	13.42 ± 1.7	13.35 ± 1.7
7078	M15	-2.15 ± 0.08	3.54 ± 0.16	15.16 ± 2.5	14.18 ± 2.3	14.11 ± 2.3
7099	M30	-2.13 ± 0.13	3.53 ± 0.14	14.96 ± 2.2	14.00 ± 2.0	13.93 ± 2.0
7492		-1.82 ± 0.30	3.61 ± 0.14	15.62 ± 2.3	14.62 ± 2.1	14.54 ± 2.1
	Ter8	-1.99 ± 0.08	3.65 ± 0.12	16.61 ± 2.0	15.52 ± 1.9	15.44 ± 1.9
	Rup106	-1.69 ± 0.05	3.15 ± 0.12	10.20 ± 1.6	10.06 ± 1.6	9.94 ± 1.5
	Pal5	-1.47 ± 0.29	3.40 ± 0.14	12.05 ± 1.8	11.36 ± 1.7	11.30 ± 1.7
	Pal12	-1.14 ± 0.20	3.30 ± 0.12	10.11 ± 1.8	9.38 ± 1.7	9.38 ± 1.7
	IC4499	-1.50 ± 0.20	3.25 ± 0.15	10.33 ± 1.6	9.78 ± 1.5	9.74 ± 1.4
	Arp2	-1.70 ± 0.11	3.29 ± 0.10	11.02 ± 1.1	10.40 ± 1.0	10.35 ± 1.0

the fact that these models fall outside the OPAL EOS tables. This question can only be answered definitively if the coverage of OPAL EOS tables is expanded so that low mass stars do not fall out of the tables. This is of some importance, as it was determined that the OPAL isochrones do not match the shape of observed colour magnitudes diagrams on the lower main sequence,

Above $M \simeq 0.7\,M_\odot$, the OPAL and Debye-Hückel stellar evolution tracks and isochrones are very similar. They predict a main-sequence turn-off magnitude which is fainter at a given age as compared to the standard isochrones. Hence, age determinations which are based on the main sequence turn-off magnitude will be systematically younger (by 0.5 – 1.5 Gyr) for the OPAL or Debye-Hückel isochrones. The greatest age reductions occur for the oldest clusters. This result has been verified by determining ΔV(TO – HB) ages for 40 globular clusters (Table 2). The OPAL and Debye-Hückel isochrones yield very similar ages, and are 6 – 7% lower than the standard isochrones.

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Figure Captions

Figure 1: The upper panel, (a), plots the run of temperature and density for $M = 0.5$ and $0.8\,M_{\odot}$, $Z = 2 \times 10^4$ stellar models near the zero-age main sequence. The solid line is for models which use the OPAL EOS, while the dashed line is for models which use the Debye-Hückel EOS. The lower panel, (b) plots the difference in density, $\Delta\rho/\rho \equiv [\rho_{\rm Debye-Hückel} - \rho_{\rm OPAL}]/\rho_{\rm Debye-Hückel}$ for $M = 0.5$ and $0.8\,M_{\odot}$ stellar models. In this plot, the run of temperature and pressure was taken from the Debye-Hückel models, so that the differences are solely due to the different EOS. The solid line at $\Delta\rho/\rho = 0$ for the $M = 0.5\,M_{\odot}$ model is due to the fact that the model is outside the OPAL table around $\log T = 5$, and the Debye-Hückel EOS was substituted.

Figure 2: The stellar evolution tracks for the $M = 0.6M_{\odot}$, $Z = 2 \times 10^4$ models with the Debye-Hückel EOS and OPAL EOS. Note that near the turn-off (where the model is entirely within the OPAL table), the two tracks nearly coincide.

Figure 3: The stellar evolution tracks for the $M = 0.8M_{\odot}$, $Z = 2 \times 10^4$ models with different EOS. The Debye-Hückel EOS and OPAL EOS evolutionary tracks are nearly identical. A model where the standard EOS was substituted for the OPAL EOS for $\log T = 4.4 - 5.2$ is shown as the short-dashed line to demonstrate the effect that falling out of the OPAL EOS has on the evolution of the model (this situation occurs for the lower mass models).

Figure 4: Isochrones with different EOSs and an age of 14 Gyr with [Fe/H] $= -1.3$ are plotted in the B, B–V plane.

Figure 5: The fit between the isochrones and the M92 fiducial sequence (solid line connecting the points) of Stetson & Harris (1988). The dashed lines are the best fitting isochrones. The left panel (a) shows the standard isochrone, while the right panel (b) plots the OPAL isochrone. The standard isochrones provide an excellent match to the data, and imply an age of 17 Gyr. The OPAL isochrones do not match the data as well.

Figure 6: Age (in Gyr) as a function of the absolute magnitude of the main sequence turn-off (M_v(TO)) for the [Fe/H] $= -1.3$ isochrones. Note that the the OPAL and Debye-Hückel isochrones are very similar, while the standard isochrones imply ages that are somewhat older, for a given M_v(TO).

Figure 7: The reduction in age implied by the OPAL EOS ($\mathrm{Age}_{\rm Standard} - \mathrm{Age}_{\rm OPAL}$) is plotted as a function of metallicity in the left panel (a), and as a function of the standard age in the right panel, (b).